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Short note

Differences between pairing and zero-range effective interactions for nuclear binding energies

K. Heyde[1,a], R. Fossion[1], J.E. García-Ramos[1,b], C. De Coster[1,c], and R.F. Casten[2]

[1] Department of Subatomic and Radiation Physics, Proeftuinstraat, 86 B-9000 Gent, Belgium
[2] Wright Nuclear Structure Laboratory, Yale University, New Haven, CT 06520, USA

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Abstract. In this paper, we show that, although the spectroscopic properties of the monopole pairing force and a zero-range delta-function interaction are very similar, their saturation properties are quite different. In particular, the predictions for binding energies when filling up a major shell are radically different past mid-shell. This has significant consequences for understanding the masses and binding energies of long isotopic chains of nuclei that will be accessible with advanced exotic beam facilities.

PACS. 21.10.Dr Binding energies and masses – 21.30.Fe Forces in hadronic systems and effective interactions – 21.60.Cs Shell model

With the advent of a wide variety of beams of exotic nuclei, the possibility of exploring the binding energies of long isotopic chains, especially on the neutron-rich side of stability can become feasible. These binding energies are determined by the shell model single-particle energies and by the residual interactions amongst the valence nucleons. One can therefore use such data to extract information on either of these important ingredients in nuclear structure.

However, to do so requires an understanding of the predictions for binding energies with different residual interactions. Two of the most important and well studied of these are the monopole pairing and the delta interactions. They have both been designed to reflect the key short-range nature of the residual interactions and to reproduce typical spectra of nuclei near closed shells.

Although the basic properties of these interactions have long been known [1–6], one of the key differences in their predictions is not well appreciated, *i.e.* the predictions of these interactions for binding energies when filling up a major shell.

We will show that, although the monopole pairing and delta forces are often considered to be similar, and do produce similar *excitation* spectra, they are fundamentally different in the way they affect particles filling a j-shell (in particular in their "saturation" properties), and they produce entirely different predictions for binding energies across a shell. So there appear subtle differences between the pairing force and the zero-range force that we wish to discuss in order to point out their consequences.

For simplicity, we consider a large single j-shell although we comment at the end on the more realistic multi-j nature of major shells. When using the monopole pairing force, and building a basis of $J^{\pi} = 0^{+}$ coupled configurations, which for seniority $v = 0$ becomes $(S_j^{\dagger})^{\frac{n}{2}}$ (with $S_j^{\dagger}$ the pair creation operator), one can evaluate the binding energy exactly with as a result (see refs. [1,3,5])

$$BE^{\text{pairing}} = G(\Omega_j - N + 1)N. \qquad (1)$$

Here, $N = \frac{n}{2}$ denotes the number of (valence) *pairs* with n describing the number of (valence) particles. We can also easily deduce the pairing force contribution to the two-neutron separation energy, which results in the expression

$$S_{2n}^{\text{pairing}} = G\left(\Omega_j + 2 - n\right) = G(\Omega_j + 2 - 2N), \qquad (2)$$

(here now, n and N refer to neutrons only, of course). These results, obtained many years ago [2] for a single-j shell, imply that with the shell fully filled with nucleons, the total binding energy is given by the diagonal expression $-G\Omega_j$. For a typical value of $G = 0.25$ MeV (using

[a] *Present address*: EP-ISOLDE, CERN, CH 1211 Geneva, Switzerland; e-mail: `kris.heyde@rug.ac.be`
[b] Visiting postdoctoral fellow of the Fund for Scientific Research-Flanders (Belgium).
[c] Postdoctoral fellow of the Fund for Scientific Research-Flanders (Belgium).

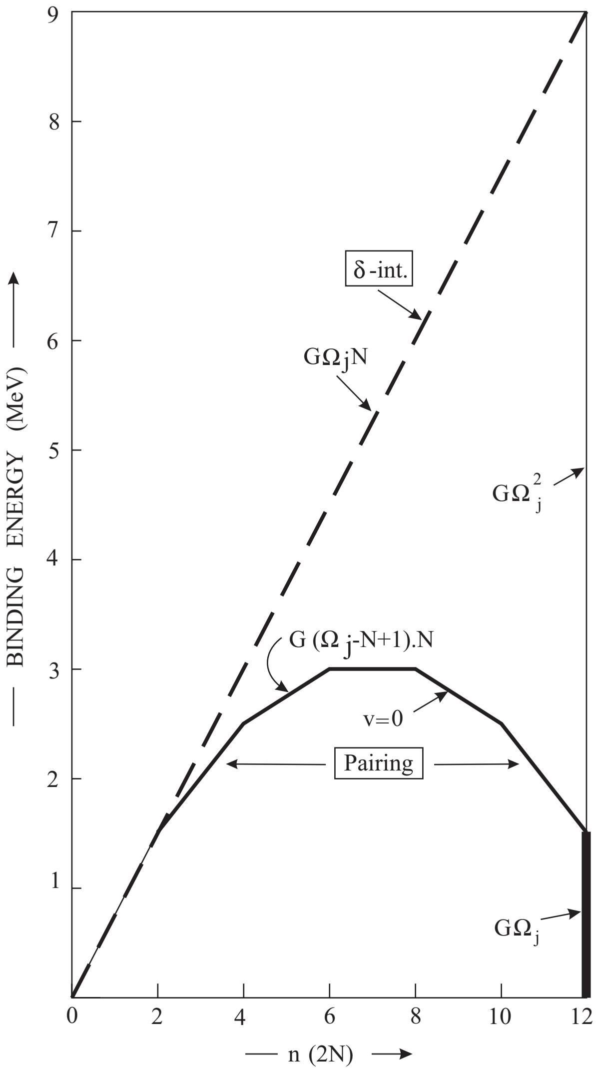

Fig. 1. Comparison of the binding energy for n particles (number of pairs $N = \frac{n}{2}$) filling the $1h_{11/2}$ orbital with $\Omega_j = 6$ using the pairing force (eq. (1)) and the zero-range delta-function force (eq. (3)).

the prescription that $G \simeq 25/A$ MeV, and for $A = 100$) and for $j = 11/2$ ($\Omega_j = 6$), this gives a rather small value of only 1.5 MeV for the total binding energy of the closed shell. This results because the pairing force counts the number of pairs moving in the j-shell in which the Pauli effect plays an essential quenching role. This result is illustrated in fig. 1 (with the solid line) where binding energies for the seniority 0^+ ($v = 0$) state in the $1h_{11/2}$ configuration is shown.

If we now use the zero-range $\delta(\vec{r}_1 - \vec{r}_2)$ force, and the same basis as we used when discussing the binding-energy effects for the pairing force, a totally different result emerges. This may seem strange at first because one has an intuitive "feeling" that pairing and zero-range forces should give rise to very much identical results. However, the zero-range interaction does not scale with the number of *pairs* as does the pairing force but scales with all the fermions that are interacting.

If we again concentrate on the ground state and thus construct the $v = 0$ state, one obtains the result [2,3]

$$\langle j^n, v = 0, JM | \sum V_{i,k} | j^n, v = 0, JM \rangle = \frac{n}{2} V_0, \qquad (3)$$

which holds for $n = 0, 2, 4, \ldots \to 2j+1$. Thus, the binding energy changes completely (see fig. 1, the dashed line), giving rise to a linear behavior in n (in contrast to the "roll-over" in the binding energies for a pairing force past mid-shell). As a result, the binding energy of the filled shell for a δ interaction amounts to $\Omega_j V_0$, where V_0 denotes the relative binding energy of the $J^\pi = 0^+$ configuration with respect to its unperturbed energy. A good estimate for this value is $V_0 = G\Omega_j$. Thus, we notice that the total binding-energy contribution for the filled shell now becomes $G{\Omega_j}^2$.

The difference between the pairing force and zero-range delta-function force thus becomes very important when one studies binding energies. Even though the detailed spectroscopic properties are not so different (the pairing force as well as the zero-range force both give rise to 0^+ ground states and to a n-independence in the energy spectra when filling up the shell-model orbital j with n identical nucleons), the use of the pairing force turns out to be a poor choice when studying binding-energy effects. The pairing force over-saturates, whereas the zero-range force perfectly saturates. That is, the binding-energy contribution for each successively added pair of nucleons decreases with n for a pairing force but is a constant for the zero-range delta interaction.

If one looks at a more realistic case, *e.g.*, filling up nucleons within the major shell between 50 and 82 nucleons, [7,8], one should consider the specific shell-model sequence of orbitals, and one cannot just put all particles in a large degenerate shell, with $j = 31/2$ (or $\Omega_j = 16$). In this case one fills in sequence, *e.g.*, the $2d_{5/2}, 1g_{7/2}, 1h_{11/2}, 2d_{3/2}$ and $3s_{1/2}$ orbitals. The arguments discussed above for a δ interaction can then be used and, as a result, one obtains a sequence of straight lines, each characterized by a slightly different slope which is fixed by the value of V_0 or by the Slater integral for that particular orbital. So we need an extra index $V_0(j)$ [2–5]. As a result, one is able to reproduce rather well the trend of increasing binding energies all through a large shell [8].

Of course, attempts to treat binding energies consistently, within a shell-model context, also have to cope with the problem of modifications in the mean field itself (varying single-particle energies ϵ_j) as a function of the filling of shells. This "monopole" issue is discussed in detail by Zuker *et al.* [9,10].

To conclude, we have discussed a simple but subtle difference characterizing the two forces that are extensively used in the study of nuclear-structure properties. Even though their relative excitation energy spectra ressemble each other very closely, important differences show up when evaluating binding energies for a filled shell-model orbital. More specifically, the total binding energy across a shell increases linearly for a δ interaction but maximizes at mid-shell and decreases past mid-shell for a monopole pairing interaction. These differences could be significant when long isochains of nuclei are studied with beams of exotic nuclei.

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